

Paper 2: Core Pure Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1(i)	$\alpha + \beta + \gamma = 8, \quad \alpha\beta + \beta\gamma + \gamma\alpha = 28, \quad \alpha\beta\gamma = 32$	B1	3.1a
	$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$	M1	1.1b
	$= \frac{7}{8}$	A1ft	1.1b
		(3)	
(ii)	$(\alpha + 2)(\beta + 2)(\gamma + 2) = (\alpha\beta + 2\alpha + 2\beta + 4)(\gamma + 2)$	M1	1.1b
	$= \alpha\beta\gamma + 2(\alpha\beta + \alpha\gamma + \beta\gamma) + 4(\alpha + \beta + \gamma) + 8$	A1	1.1b
	$= 32 + 2(28) + 4(8) + 8 = 128$	A1	1.1b
		(3)	
	Alternative:		
	$(x - 2)^3 - 8(x - 2)^2 + 28(x - 2) - 32 = 0$	M1	1.1b
	$= \dots - 8 + \dots - 32 + \dots - 56 - 32 = -128$	A1	1.1b
	$\therefore (\alpha + 2)(\beta + 2)(\gamma + 2) = 128$	A1	1.1b
		(3)	
(iii)	$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$	M1	3.1a
	$= 8^2 - 2(28) = 8$	A1ft	1.1b
		(2)	
(8 marks)			
Notes:			
(i)			
B1: Identifies the correct values for all 3 expressions (can score anywhere)			
M1: Uses a correct identity			
A1ft: Correct value (follow through their 8, 28 and 32)			
(ii)			
M1: Attempts to expand			
A1: Correct expansion			
A1: Correct value			
Alternative:			
M1: Substitutes $x - 2$ for x in the given cubic			
A1: Calculates the correct constant term			
A1: Changes sign and so obtains the correct value			
(iii)			
M1: Establishes the correct identity			
A1ft: Correct value (follow through their 8, 28 and 32)			

Question	Scheme	Marks	AOs
2(a)	$\begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 2 \\ 12 \end{pmatrix} = 18 - 8 + 24$	M1	3.1a
	$d = \frac{18 - 8 + 24 - 5}{\sqrt{3^2 + 4^2 + 2^2}}$	M1	1.1b
	$= \sqrt{29}$	A1	1.1b
		(3)	
(b)	$\begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} = \dots$ and $\begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} = \dots$	M1	2.1
	$\begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} = 0$ and $\begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} = 0$ $\therefore -\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ is perpendicular to Π_2	A1	2.2a
		(2)	
(c)	$\begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} = -3 + 12 + 2$	M1	1.1b
	$\sqrt{(-1)^2 + (-3)^2 + 1^2} \sqrt{(3)^2 + (-4)^2 + 2^2} \cos \theta = 11$ $\Rightarrow \cos \theta = \frac{11}{\sqrt{(-1)^2 + (-3)^2 + 1^2} \sqrt{(3)^2 + (-4)^2 + 2^2}}$	M1	2.1
	So angle between planes $\theta = 52^\circ$ *	A1*	2.4
		(3)	
(8 marks)			
Notes:			
(a)			
M1: Realises the need to and so attempts the scalar product between the normal and the position vector			
M1: Correct method for the perpendicular distance			
A1: Correct distance			
(b)			
M1: Recognises the need to calculate the scalar product between the given vector and both direction vectors			
A1: Obtains zero both times and makes a conclusion			
(c)			
M1: Calculates the scalar product between the two normal vectors			
M1: Applies the scalar product formula with their 11 to find a value for $\cos \theta$			
A1*: Identifies the correct angle by linking the angle between the normal and the angle between the planes			

Question	Scheme	Marks	AOs
3(i)(a)	$ \mathbf{M} = 2(1+2) - a(-1-1) + 4(2-1) = 0 \Rightarrow a = \dots$	M1	2.3
	The matrix $\mathbf{M}$ has an inverse when $a \neq -5$	A1	1.1b
		(2)	
(b)	Minors : $\begin{pmatrix} 3 & -2 & 1 \\ -a-8 & 2 & a+4 \\ 4-a & -6 & -2-a \end{pmatrix}$ or Cofactors : $\begin{pmatrix} 3 & 2 & 1 \\ a+8 & 2 & -a-4 \\ 4-a & 6 & -2-a \end{pmatrix}$	B1	1.1b
	$\mathbf{M}^{-1} = \frac{1}{ \mathbf{M} } \text{adj}(\mathbf{M})$	M1	1.1b
	$\mathbf{M}^{-1} = \frac{1}{2a+10} \begin{pmatrix} 3 & a+8 & 4-a \\ 2 & 2 & 6 \\ 1 & -a-4 & -2-a \end{pmatrix}$	2 correct rows or columns. Follow through their det $\mathbf{M}$	A1ft 1.1b
		All correct. Follow through their det $\mathbf{M}$	A1ft 1.1b
		(4)	
(ii)	When $n = 1$, $\text{lhs} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$, $\text{rhs} = \begin{pmatrix} 3^1 & 0 \\ 3(3^1-1) & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$ So the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^k = \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$	M1	2.4
	$\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$	M1	2.1
	$= \begin{pmatrix} 3 \times 3^k & 0 \\ 3 \times 3(3^k-1) + 6 & 1 \end{pmatrix}$	A1	1.1b
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 3(3^{k+1}-1) & 1 \end{pmatrix}$	A1	1.1b
	If the statement is true for $n = k$ then it has been shown true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all positive integers n	A1	2.4
		(6)	
(12 marks)			

Question 3 notes:**(i)(a)**

M1: Attempts determinant, equates to zero and attempts to solve for a in order to establish the restriction for a

A1: Provides the correct condition for a if **M** has an inverse

(i)(b)

B1: A correct matrix of minors or cofactors

M1: For a complete method for the inverse

A1ft: Two correct rows following through their determinant

A1ft: Fully correct inverse following through their determinant

(ii)

B1: Shows the statement is true for $n = 1$

M1: Assumes the statement is true for $n = k$

M1: Attempts to multiply the correct matrices

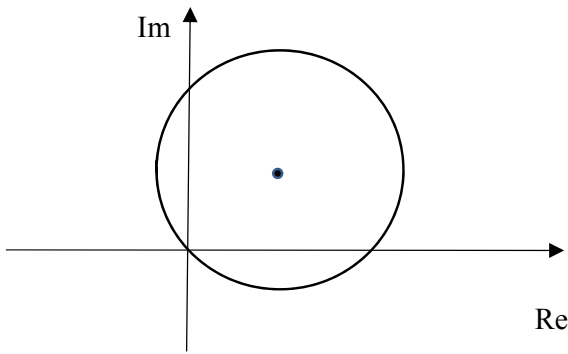
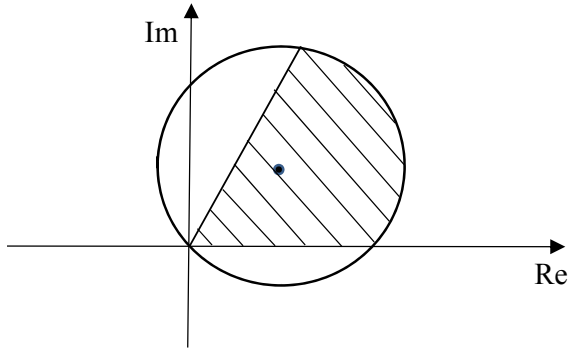
A1: Correct matrix in terms of k

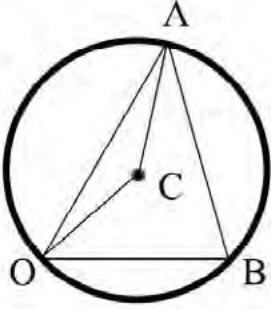
A1: Correct matrix in terms of $k + 1$

A1: Correct complete conclusion

Question	Scheme	Marks	AOs
4(a)	$z^n + z^{-n} = \cos n\theta + i \sin n\theta + \cos n\theta - i \sin n\theta$	M1	2.1
	$= 2 \cos n\theta^*$	A1*	1.1b
		(2)	
(b)	$(z + z^{-1})^4 = 16 \cos^4 \theta$	B1	2.1
	$(z + z^{-1})^4 = z^4 + 4z^2 + 6 + 4z^{-2} + z^{-4}$	M1	2.1
	$= z^4 + z^{-4} + 4(z^2 + z^{-2}) + 6$	A1	1.1b
	$= 2 \cos 4\theta + 4(2 \cos 2\theta) + 6$	M1	2.1
	$\cos^4 \theta = \frac{1}{8}(\cos 4\theta + 4 \cos 2\theta + 3)^*$	A1*	1.1b
		(5)	
(7 marks)			
Notes:			
(a)			
M1: Identifies the correct form for z^n and z^{-n} and adds to progress to the printed answer			
A1*: Achieves printed answer with no errors			
(b)			
B1: Begins the argument by using the correct index with the result from part (a)			
M1: Realises the need to find the expansion of $(z + z^{-1})^4$			
A1: Terms correctly combined			
M1: Links the expansion with the result in part (a)			
A1*: Achieves printed answer with no errors			

Question	Scheme	Marks	AOs
5(a)	$\frac{dy}{dx} = \sin x \cosh x + \cos x \sinh x$	M1	1.1a
	$\frac{d^2y}{dx^2} = \cos x \cosh x + \sin x \sinh x + \cos x \cosh x - \sin x \sinh x$ $(= 2 \cos x \cosh x)$	M1	1.1b
	$\frac{d^3y}{dx^3} = 2 \cos x \sinh x - 2 \sin x \cosh x$	M1	1.1b
	$\frac{d^4y}{dx^4} = -4 \sinh x \sin x = -4y^*$	A1*	2.1
		(4)	
(b)	$\left(\frac{d^2y}{dx^2}\right)_0 = 2, \left(\frac{d^6y}{dx^6}\right)_0 = -8, \left(\frac{d^{10}y}{dx^{10}}\right)_0 = 32$	B1	3.1a
	Uses $y = y_0 + xy'_0 + \frac{x^2}{2!}y''_0 + \frac{x^3}{3!}y'''_0 + \dots$ with their values	M1	1.1b
	$= \frac{x^2}{2!}(2) + \frac{x^6}{6!}(-8) + \frac{x^{10}}{10!}(32)$	A1	1.1b
	$= x^2 - \frac{x^6}{90} + \frac{x^{10}}{113400}$	A1	1.1b
		(4)	
(c)	$2(-4)^{n-1} \frac{x^{4n-2}}{(4n-2)!}$	M1 A1	3.1a 2.2a
		(2)	
(10 marks)			
Notes:			
(a)			
M1: Realises the need to use the product rule and attempts first derivative			
M1: Realises the need to use a second application of the product rule and attempts the second derivative			
M1: Correct method for the third derivative			
A1*: Obtains the correct 4 th derivative and links this back to y			
(b)			
B1: Makes the connection with part (a) to establish the general pattern of derivatives and finds the correct non-zero values			
M1: Correct attempt at Maclaurin series with their values			
A1: Correct expression un-simplified			
A1: Correct expression and simplified			
(c)			
M1: Generalising, dealing with signs, powers and factorials			
A1: Correct expression			

Question	Scheme	Marks	AOs
6(a)(i)		M1	1.1b
		A1	1.1b
(a)(ii)	$ z - 4 - 3i = 5 \Rightarrow x + iy - 4 - 3i = 5 \Rightarrow (x - 4)^2 + (y - 3)^2 = \dots$	M1	2.1
	$(x - 4)^2 + (y - 3)^2 = 25$ or any correct form	A1	1.1b
	$(r \cos \theta - 4)^2 + (r \sin \theta - 3)^2 = 25$ $\Rightarrow r^2 \cos^2 \theta - 8r \cos \theta + 16 + r^2 \sin^2 \theta - 6r \sin \theta + 9 = 25$ $\Rightarrow r^2 - 8r \cos \theta - 6r \sin \theta = 0$	M1	2.1
	$\therefore r = 8 \cos \theta + 6 \sin \theta^*$	A1*	2.2a
		(6)	
(b)(i)		B1	1.1b
		B1ft	1.1b
(b)(ii)	$A = \frac{1}{2} \int r^2 d\theta = \frac{1}{2} \int (8 \cos \theta + 6 \sin \theta)^2 d\theta$ $= \frac{1}{2} \int (64 \cos^2 \theta + 96 \sin \theta \cos \theta + 36 \sin^2 \theta) d\theta$	M1	3.1a
	$= \frac{1}{2} \int (32(\cos 2\theta + 1) + 96 \sin \theta \cos \theta + 18(1 - \cos 2\theta)) d\theta$	M1	1.1b
	$= \frac{1}{2} \int (14 \cos 2\theta + 50 + 48 \sin 2\theta) d\theta$	A1	1.1b
	$= \frac{1}{2} [7 \sin 2\theta + 50\theta - 24 \cos 2\theta]_0^{\frac{\pi}{3}} = \frac{1}{2} \left\{ \left(\frac{7\sqrt{3}}{2} + \frac{50\pi}{3} + 12 \right) - (-24) \right\}$	M1	2.1
	$= \frac{7\sqrt{3}}{4} + \frac{25\pi}{3} + 18$	A1	1.1b
		(7)	

Question	Scheme	Marks	AOs
	(b)(ii) Alternative:  Candidates may take a geometric approach e.g. by finding sector + 2 triangles		
	Angle $ACB = \left(\frac{2\pi}{3}\right)$ so area sector $ACB = \frac{1}{2}(5)^2 \frac{2\pi}{3}$ Area of triangle $OCB = \frac{1}{2} \times 8 \times 3$	M1	3.1a
	Sector area ACB + triangle area $OCB = \frac{25\pi}{3} + 12$	A1	1.1b
	Area of triangle OAC: Angle $ACO = 2\pi - \frac{2\pi}{3} - \cos^{-1}\left(\frac{5^2 + 5^2 - 8^2}{2 \times 5 \times 5}\right)$ so area $OAC = \frac{1}{2}(5)^2 \sin\left(\frac{4\pi}{3} - \cos^{-1}\left(\frac{-7}{25}\right)\right)$	M1	1.1b
	$= \frac{25}{2} \left(\sin \frac{4\pi}{3} \cos\left(\cos^{-1}\left(\frac{-7}{25}\right)\right) - \cos \frac{4\pi}{3} \sin\left(\cos^{-1}\left(\frac{-7}{25}\right)\right) \right)$ $= \frac{25}{2} \left(\left(\frac{7\sqrt{3}}{50}\right) + \frac{1}{2} \sqrt{1 - \left(\frac{7}{25}\right)^2} \right) = \frac{7\sqrt{3}}{4} + 6$ Total area = $\frac{25\pi}{3} + \frac{1}{2} \times 8 \times 3 + 6 + \frac{7\sqrt{3}}{4}$	M1	2.1
	$= \frac{7\sqrt{3}}{4} + \frac{25\pi}{3} + 18$	A1	1.1b
(13 marks)			

Question 6 notes:	
(a)(i)	
M1:	Draws a circle which passes through the origin
A1:	Fully correct diagram
(a)(ii)	
M1:	Uses $z = x + iy$ in the given equation and uses modulus to find equation in x and y only
A1:	Correct equation in terms of x and y in any form – may be in terms of r and θ
M1:	Introduces polar form, expands and uses $\cos^2 \theta + \sin^2 \theta = 1$ leading to a polar equation
A1*:	Deduces the given equation (ignore any reference to $r = 0$ which gives a point on the curve)
(b)(i)	
B1:	Correct pair of rays added to their diagram
B1ft:	Area between their pair of rays and inside their circle from (a) shaded, as long as there is an intersection
(b)(ii)	
M1:	Selects an appropriate method by linking the diagram to the polar curve in (a), evidenced by use of the polar area formula
M1:	Uses double angle identities
A1:	Correct integral
M1:	Integrates and applies limits
A1:	Correct area
(b)(ii) Alternative:	
M1:	Selects an appropriate method by finding angle ACB and area of sector ACB and finds area of triangle OCB to make progress towards finding the required area
A1:	Correct combined area of sector ACB + triangle OCB
M1:	Starts the process of finding the area of triangle OAC by calculating angle ACO and attempts area of triangle OAC
M1:	Uses the addition formula to find the exact area of triangle OAC and employs a full correct method to find the area of the shaded region
A1:	Correct area

Question	Scheme	Marks	AOs
7(a)	$r = 10 \frac{df}{dt} - 2f \Rightarrow \frac{dr}{dt} = 10 \frac{d^2f}{dt^2} - 2 \frac{df}{dt}$	M1	2.1
	$10 \frac{d^2f}{dt^2} - 2 \frac{df}{dt} = -0.2f + 0.4 \left(10 \frac{df}{dt} - 2f \right)$	M1	2.1
	$\frac{d^2f}{dt^2} - 0.6 \frac{df}{dt} + 0.1f = 0^*$	A1*	1.1b
		(3)	
(b)	$m^2 - 0.6m + 0.1 = 0 \Rightarrow m = \frac{0.6 \pm \sqrt{0.6^2 - 4 \times 0.1}}{2}$	M1	3.4
	$m = 0.3 \pm 0.1i$	A1	1.1b
	$f = e^{\alpha t} (A \cos \beta t + B \sin \beta t)$	M1	3.4
	$f = e^{0.3t} (A \cos 0.1t + B \sin 0.1t)$	A1	1.1b
		(4)	
(c)	$\frac{df}{dt} = 0.3e^{0.3t} (A \cos 0.1t + B \sin 0.1t) + 0.1e^{0.3t} (B \cos 0.1t - A \sin 0.1t)$	M1	3.4
	$r = 10 \frac{df}{dt} - 2f$ $= e^{0.3t} ((3A+B) \cos 0.1t + (3B-A) \sin 0.1t) - 2e^{0.3t} (A \cos 0.1t + B \sin 0.1t)$	M1	3.4
	$r = e^{0.3t} ((A+B) \cos 0.1t + (B-A) \sin 0.1t)$	A1	1.1b
		(3)	
(d)(i)	$t = 0, f = 6 \Rightarrow A = 6$	M1	3.1b
	$t = 0, r = 20 \Rightarrow B = 14$	M1	3.3
	$r = e^{0.3t} (20 \cos 0.1t + 8 \sin 0.1t) = 0$	M1	3.1b
	$\tan 0.1t = -2.5$	A1	1.1b
	2019	A1	3.2a
(d)(ii)	3750 foxes	B1	3.4
(d)(iii)	e.g. the model predicts a large number of foxes are on the island when the rabbits have died out and this may not be sensible	B1	3.5a
		(7)	
(17 marks)			

Question 7 notes:	
(a)	M1: Attempts to differentiate the first equation with respect to t M1: Proceeds to the printed answer by substituting into the second equation A1*: Achieves the printed answer with no errors
(b)	M1: Uses the model to form and solve the auxiliary equation A1: Correct values for m M1: Uses the model to form the CF A1: Correct CF
(c)	M1: Differentiates the expression for the number of foxes M1: Uses this result to find an expression for the number of rabbits A1: Correct equation
(d)(i)	M1: Realises the need to use the initial conditions in the model for the number of foxes M1: Realises the need to use the initial conditions in the model for the number of rabbits to find both unknown constants M1: Obtains an expression for r in terms of t and sets $= 0$ A1: Rearranges and obtains a correct value for $\tan$ A1: Identifies the correct year
(d)(ii)	B1: Correct number of foxes
(d)(iii)	B1: Makes a suitable comment on the outcome of the model